

Markov Chain Model for Probability of Weekly Rainfall in Mandya District, Karnataka

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ABSTRACT: It is necessary to know the sequence of dry and wet periods for successful agricultural management and planning of soil and water conservation measures. Probability analysis is a very useful tool for making important decisions in agricultural operations. In this study, Markov chain model has been extensively used to study spell distribution. For this purpose a week period was considered as the optimum length of time. The present study has been carried out to find the probabilities of occurrence of wet week (W), wet week preceded by wet week (W/W) and wet preceded by dry week (W/D) at a threshold rainfall limit of 5 mm/week. For the analysis, Daily rainfall data was converted to weekly period for 30 years from 1984 – 2013 for Mandya district of Karnataka. The results reveal that, the probability of occurrence of wet weeks during the pre monsoon season (18th SMW to 23rd SMW) was around 20% and during monsoon season (35th SMW to 46th SMW) it was around 35%. Conditional probability of wet week followed by wet had the similar pattern as that of initial probability of wet weeks. Conditional probability of wet given wet weeks was around 30% during pre monsoon season. However, during the monsoon season, it was found to be more than 50% (36th SMW and 39th SMW to 43rd SMW). The conditional probability of wet given dry weeks was found to be below 30% during the pre monsoon and monsoon seasons.

KEYWORDS: Standard meteorological weeks (SMW), Markov chain model, Dry and Wet spells

I. INTRODUCTION

The yield of crops particularly under rainfed condition depends on the rainfall pattern. A simple criteria related to sequential phenomena like dry and wet spells could be used for analyzing rainfall data to obtain specific information needed for crop planning and for carrying out agricultural operations, Markov chain probability model has been recognized as a suitable model to explain the long term frequency behaviour of wet or dry spells. Several authors have demonstrated its practical utility in agricultural planning for both long and short term periods. This model enables to determine the probability of occurrence of dry and wet spells during a particular week.

Mandya district with a geographical area of 4,98,244 hectares lies in the Southern part of Karnataka state between 76° 19' and 77° 20' East Longitude and 12° 13' and 13° 04' North Latitude. The population is about 19.25 lakhs. Its district comprises of seven taluks, thirty one Hoblis and 1,369 villages. The district is one of the most agriculturally prosperous districts in Karnataka. With the advent of irrigation from the K.R.Sagar reservoir (built during 1930's), there has been a substantially marked transformation in cropping pattern, composition of crops, better grown yield level, ultimately leading to better economic conditions of the people. The total irrigated area is 1,16,901 hectares out of which around 88,000 hectares is being irrigated by K.R.Sagar and around 16,000 hectares by Hemavathi reservoir. The rest of the land is irrigated by other sources like tanks, wells and bore wells.

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The major crops of the Mandya district are ragi (grown in 85,467 ha.), rice (grown in 79,892 ha.) sugarcane (grown in 30,630 ha.), pulses (Horse gram, Tur, Cowpea, Green gram, Black gram, Avare) and Oilseeds (groundnut and sesame). Agriculture in the district depends on rainfall, river, well and tank irrigation.

Majority of geographical area of Mandya district is covered by Cauvery basin and agriculture in the district is classified under agro- climatic zone 6 i.e. Southern dry zone. The crop sowing periods are early kharif, late kharif and summer. The average annual rainfall is 700mm. The district gets a bimodal distribution of rainfall. Generally, the first peak is during April, May and the second during September, October. The month of June is known for dry spell. The district is blessed with irrigation from two major reservoirs, K.R. Sagar and Hemavathi. Other than these, there are number of anecut channels. Hemavathi left bank canal is irrigating parts of K.R. Pet, Nagamangala, Pandavapura, and Mandya taluks. About 48% of the total area in the district is irrigated from all source of irrigation.

Markov Chain is generally recognized as a simple and effective description of the rainfall occurrence. The amount and pattern of rainfall are among the most important weather characteristics and they affect agriculture profoundly. In addition to their direct effects on water balance in soil, they are strongly related to other weather variables such as solar radiation, temperature, and humidity, which are also important factors affecting the growth and development of crops, pests, diseases and weeds (Hossain and Anam, 2012). The use of Markov chain models to define probable yield of dry and wet spells has been well recognized by various authors starting from the pioneering work of Gabriel and Neumann (1962). Further, Markov process has been widely used for rainfall analysis by Caskey (1963), Weiss (1964), Basu (1971), Bhargava et al. (1973), Katz (1974), Sundararaj and Ramachandra (1975), Aneja and Srivastava (1986), Rahman (1999), Ravindranan and Dani (1993), Akhtar and Hossain (1998), Ajayi and Olufayu (2002), Banik et al. (2002), Rahman and Basher Mian (2002), Srinivasareddy et al (2008), Piyadasa and Sonnadara, (2010), Amro and Nassir (2011), Soomro and Mir (2011), Hossain and Anam (2012), Priyaranjan(2012), Senthilvelan et al. (2012), Mangaraj et al. (2013), Admasu et al (2014) and among others. In this paper, a 2-state Markov Chain probability model has been used to identify the pattern of occurrence of rainfall in Mandya district.

II. MATERIAL AND METHODS

For the present study, the daily rainfall data of 32 rain gauge stations located in seven taluks of Mandya district were collected from the Agro-meteorological centre, G.K.V.K, UAS, Bengaluru for a period of 30 years from 1984 to 2013.

Markov chain model

A Wet week (or a Dry week) has been defined as one with ≥ 5 mm (≤ 5 mm) of rainfall according to definition proposed by the Indian Meteorological Department. This gives a sequence of wet and dry weeks. Further, under the assumption that the occurrence of a wet or a dry week is influenced only by the weather condition of the previous week, the process of occurrence of wet and dry weeks can be described by a 2- state Markov chain with wet and dry weeks as the two states. The transition probability matrix P , which describes the 2 – state Markov chain model is given by,

$$P = \begin{bmatrix} P_{D/D} & P_{D/W} \\ P_{W/D} & P_{W/W} \end{bmatrix},$$

with $P_{D/D} + P_{D/W} = 1$ and $P_{W/D} + P_{W/W} = 1$, where $P_{D/D}$, $P_{D/W}$, $P_{W/D}$ and $P_{W/W}$ are the transition probabilities. That is, they are respectively the probabilities of the following conditional events:

$E_{D/D}$: A week is a dry week given that the preceding week was a dry week.

$E_{W/D}$: A week is a wet week given that the preceding week was a dry week.

$E_{D/W}$: A week is a dry week given that the preceding week was a wet week.

$E_{W/W}$: A week is a wet week given that the preceding week was a wet week.

Suppose that each week from January to December is classified according to the occurrence of the four events $E_{D/D}$, $E_{D/W}$, $E_{W/D}$ and $E_{W/W}$ such that 1st week depends on the 52nd week of December. Then, repeating this process for each year, frequencies of the occurrences of events are counted. Let these observed frequencies be denoted a, b, c and d

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for the respective events with $a + b = n_0$ and $c + d = n_1$. The maximum likelihood estimates of the unknown probabilities $P_{D/D}$, $P_{D/W}$, $P_{W/D}$ and $P_{W/W}$ i.e., the parameters of the model are obtained as,

$$\hat{P}_{D/D} = p_{D/D} = \frac{a}{n_0}, \quad \hat{P}_{D/W} = p_{D/W} = \frac{b}{n_0}$$

$$\hat{P}_{W/D} = p_{W/D} = \frac{c}{n_1}, \quad \hat{P}_{W/W} = p_{W/W} = \frac{d}{n_1}$$

The transition probabilities are conditional probabilities. But, the probability of a dry week (P_D) and the probability of wet week (P_W) are estimated from the observed frequencies of the conditional events as follows:

$$\hat{P}_D = p_D = \frac{a + c}{n_0 + n_1} \text{ and } \hat{P}_W = p_W = \frac{b + d}{n_0 + n_1}$$

These unconditional probabilities are also called binomial probabilities treating a wet week as a success and a dry week as a failure.

III. RESULTS AND DISCUSSION

The descriptive statistics of annual rainfall of Mandya district for the study period is presented in Table 1. The minimum rainfall ranged between 112 mm to 462.2 mm for 2012 and

Table 1: Descriptive statistics of Annual rainfall in Mandya district from 1984-2013

Year	Annual Rainfall (mm)					
	Min	Max	Mean	SD	CV (%)	Skewness
1984	207.0	1189.1	604.0	211.9	35.1	0.7
1985	134.4	1153.2	512.6	218.1	42.5	1.1
1986	309.3	1199.8	709.6	213.9	30.1	0.3
1987	179.6	1288.0	747.4	211.7	28.3	0.2
1988	128.8	973.1	661.3	185.2	28.0	-0.9
1989	231.6	1273.0	623.7	209.9	33.6	0.6
1990	142.8	952.0	434.6	167.5	38.5	0.9
1991	227.6	1170.6	789.8	225.5	28.5	-0.6
1992	273.2	906.4	601.2	175.7	29.2	-0.8
1993	328.4	1170.7	693.8	189.3	27.3	-0.1
1994	234.0	962.8	659.3	183.9	27.9	-0.2
1995	295.1	1216.4	682.2	209.6	30.7	0.7
1996	263.0	1051.2	654.3	201.9	30.8	-0.01
1997	284.2	1238.6	668.1	218.6	32.7	0.4
1998	277.2	1271.9	710.1	223.3	31.4	0.2
1999	358.8	1277.2	778.5	225.5	28.9	0.4
2000	341.8	1319.7	845.7	249.6	29.5	-0.4
2001	315.0	931.2	701.1	145.1	20.7	-0.8
2002	206.2	1050.1	538.3	192.7	35.8	1.1
2003	212.0	1199.5	597.7	230.9	38.6	0.8

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2004	129.0	1474.6	788.9	239.7	30.4	-0.4
2005	462.2	1403.1	944.1	289.5	30.6	-0.3
2006	152.5	1311.5	523.6	233.3	44.5	1.3
2007	332.7	1021.8	689.0	164.0	23.8	-0.03
2008	287.7	1099.6	708.2	197.2	27.8	0.05
2009	288.6	909.8	677.9	162.5	23.9	-0.8
2010	382.0	1179.0	854.6	205.5	24.0	-0.6
2011	212.0	1019.0	658.3	157.4	23.9	-0.2
2012	112.0	677.0	445.7	135.9	30.5	-0.3
2013	213.0	814.0	564.9	139.8	24.7	-0.8

2005 respectively.. The minimum among the maximum rainfall was 617 mm during 2012 while the maximum was 1474.6 mm during the year 2004. The average rainfall varied between 434.6 mm to 944.1 mm during 1990 and 2005 respectively. The coefficient of variation was minimum (20.7%) during 2001 and was maximum (44.5%) during 2006. Also from 2007 onwards the skewness was observed to be negative due to the fact the average rainfall was below the modal value.

Probabilities of Wet and Dry spells

The daily rainfall data was converted to Standard Meteorological Week (SMW) and based on the quantum of rainfall (5mm), the SMW's were classified as dry and wet spells to study its pattern. A two state Markov chain model was used to evaluate the marginal and conditional probabilities of dry and wet spells by considering less than 5 mm rainfall in a week as a dry week and 5 mm or more as a wet week.

The marginal probabilities of wet weeks $P_{(w)}$, conditional probabilities of wet week followed by dry week $P_{(w/w)}$ and wet week followed by wet week $P_{(w/D)}$ for the period from 1984 to 2013 were computed and are presented in Table 2.

The results from Table clearly reveals that, the probability of occurrence of wet weeks $P_{(w)}$ during the pre monsoon season (18th SMW to 23rd SMW) was around 20% and during monsoon (35th SMW to 46rd SMW) it was around 35%. Conditional probability of wet week followed by wet week $P_{(w/w)}$ had the similar pattern as that of initial probability of wet weeks. Conditional probability of wet given wet weeks $P_{(w/w)}$ was around 30% during pre monsoon season. However, during the monsoon season it was found to be more than 50% (36th SMW and 39th SMW to 43rd SMW). The conditional probability of wet given dry weeks $P_{(w/D)}$ was found to be below 30% during the pre monsoon and monsoon seasons. The probabilities of standard weeks being wet are presented in the figure. The difference between $P_{(w/w)}$ and $P_{(w/D)}$ decreased as the monsoon season progressed suggesting that the probabilities of a week being wet became less dependent on the condition of the previous week as the rains became more steady. Similarly, the difference between $P_{(D/D)}$ and $P_{(D/w)}$ decreased as the monsoon season progressed corroborating the observation that the probabilities of having a wet week became less dependent on the condition of the previous week as the rains became more steady. However, the difference between $P_{(D/D)}$ and $P_{(D/w)}$ increased during the monsoon season to show that the conditional dependence of dry weeks was inevitable even during the monsoon seasonal months of June to October in Mandya district. This clearly indicates that pre monsoon rain can be utilized for short duration crops like pulses and during monsoon season long duration crops like rice and sugarcane can be taken up.

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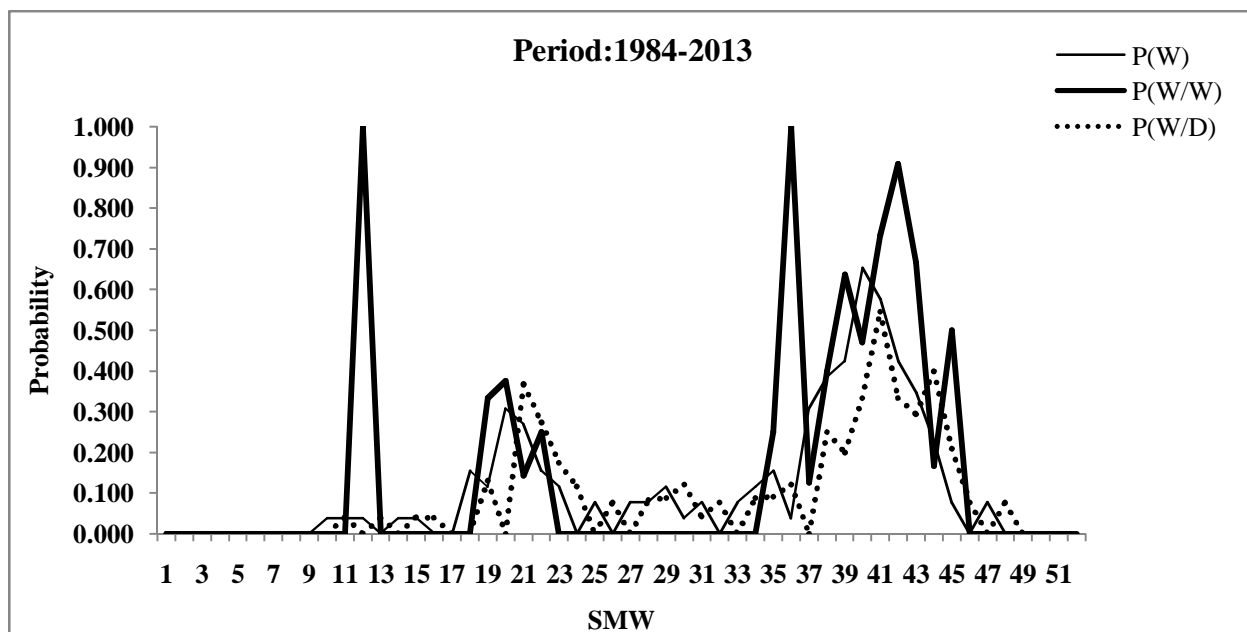


Fig.: Probability of standard week being wet for Mandya district

IV. CONCLUSION

The analysis of consecutive dry and wet spells during monsoon season reveal that there are more than 50% chances that 2 consecutive wet weeks may occur during 36th SMW and 39th SMW to 43rd SMW. Similarly, during pre monsoon season, viz., 18th SMW to 23rd SMW, the probabilities of occurrence of 2 consecutive wet weeks were around 30%. The probabilities of week being wet were found to be around 20% and 35% during pre monsoon and monsoon seasons respectively. Further, the chance of wet followed by dry week was below 30% during pre monsoon and monsoon seasons respectively.

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Table 2: Marginal, Joint and Conditional probabilities of Rainfall during 1984-2013 (Limit = 5mm/ week)

	Joint Probability				Conditional Probability				SMW	Marginal probability		Joint Probability				Conditional Probability	
	$P_{(DD)}$	$P_{(DW)}$	$P_{(WW)}$	$P_{(WD)}$	$P_{(D/D)}$	$P_{(D/W)}$	$P_{(W/W)}$	$P_{(W/D)}$		$P_{(D)}$	$P_{(W)}$	$P_{(DD)}$	$P_{(DW)}$	$P_{(WW)}$	$P_{(WD)}$	$P_{(D/D)}$	$P_{(D/W)}$
0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	27	0.9	0.1	0.9	0.1	0.0	0.0	1.0	1.0
0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	28	0.9	0.1	0.8	0.1	0.0	0.1	0.9	1.0
0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	29	0.9	0.1	0.8	0.1	0.0	0.1	0.9	1.0
0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	30	1.0	0.0	0.8	0.0	0.0	0.1	0.9	1.0
0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	31	0.9	0.1	0.9	0.1	0.0	0.0	1.0	1.0
0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	32	1.0	0.0	0.9	0.0	0.0	0.1	0.9	0.0
0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	33	0.9	0.1	0.9	0.1	0.0	0.0	1.0	1.0
0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	34	0.9	0.1	0.8	0.1	0.0	0.1	0.9	1.0
0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	35	0.8	0.2	0.8	0.1	0.0	0.1	0.9	0.8
0	1.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	36	1.0	0.0	0.8	0.0	0.0	0.1	0.9	0.0
0	0.9	0.0	0.0	0.0	1.0	1.0	0.0	0.0	37	0.7	0.3	0.7	0.3	0.0	0.0	1.0	0.9
0	1.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	38	0.6	0.4	0.5	0.2	0.2	0.2	0.8	0.6
0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	39	0.6	0.4	0.5	0.2	0.3	0.1	0.8	0.4
0	1.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	40	0.3	0.7	0.2	0.3	0.3	0.1	0.7	0.5
0	0.9	0.0	0.0	0.0	1.0	1.0	0.0	0.0	41	0.4	0.6	0.2	0.2	0.4	0.2	0.5	0.3
0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	42	0.6	0.4	0.4	0.0	0.4	0.2	0.7	0.1
0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	43	0.7	0.3	0.5	0.1	0.2	0.2	0.7	0.3
2	0.8	0.2	0.0	0.0	1.0	1.0	0.0	0.0	44	0.8	0.2	0.5	0.2	0.0	0.3	0.6	0.8
1	0.8	0.1	0.0	0.1	0.9	0.7	0.3	0.1	45	0.9	0.1	0.7	0.0	0.0	0.2	0.8	0.5
3	0.7	0.2	0.1	0.0	1.0	0.6	0.4	0.0	46	1.0	0.0	0.9	0.0	0.0	0.1	0.9	0.0
3	0.5	0.2	0.0	0.3	0.6	0.9	0.1	0.4	47	0.9	0.1	0.9	0.1	0.0	0.0	1.0	1.0
2	0.6	0.1	0.0	0.2	0.7	0.8	0.3	0.3	48	1.0	0.0	0.9	0.0	0.0	0.1	0.9	0.0
1	0.7	0.1	0.0	0.2	0.8	1.0	0.0	0.2	49	1.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0
0	0.9	0.0	0.0	0.1	0.9	0.0	0.0	0.1	50	1.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0
1	0.9	0.1	0.0	0.0	1.0	1.0	0.0	0.0	51	1.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0
0	0.9	0.0	0.0	0.1	0.9	0.0	0.0	0.1	52	1.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0

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